

## OCEAN /ATMOSPHERIC PROCESSES FORMULA SHEET

$$\rho = \frac{P}{R_d T}$$

$$T_v = T(1 + 0.61r)$$

$$\partial P / \partial z = -\rho g$$

$$P = P_o e^{\frac{-gz}{RT}}$$

$$dH = \rho c_p dT - dP$$

$$\Gamma_{ad} \equiv -(\partial T / \partial z)_{ad} = g / c_p$$

$$T_0 = T(P_0 / P)^{R/c_p}$$

$$S = \frac{g}{\theta_v} \frac{\partial \theta_v}{\partial z}$$

$$E = -\frac{1}{\rho} \frac{\partial \sigma_{t,\theta}}{\partial z}$$

$$\frac{Dw}{Dt} = -g - \frac{1}{\rho} \frac{\partial P_e}{\partial z} = -g + \frac{\rho_e g}{\rho} = g \left( \frac{\rho_e - \rho}{\rho} \right) = g \left( \frac{\theta_v - \theta_{ve}}{\theta_{ve}} \right)$$

$$N = (gE)^{1/2} (\text{rad} / \text{s})$$

$$T = \frac{2\pi}{N} (s)$$

$$N = (S)^{1/2} (\text{rad} / \text{s})$$

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$$R_e = \frac{UL}{v}$$

$$R_i = \frac{N^2}{\left( \frac{\partial U}{\partial z} \right)^2}$$

$$\frac{DQ}{Dt} = \frac{\partial Q}{\partial t} + \vec{V} \bullet \vec{\nabla} Q$$

$$\frac{1}{\rho} \frac{D\rho}{Dt} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{1}{\rho} \frac{D\rho}{Dt} + \vec{\nabla} \bullet \vec{V} = 0$$

$$w \frac{\partial (\ln(\rho_o))}{\partial z} + \vec{\nabla} \bullet \vec{V} = \vec{\nabla} \bullet \rho_o \vec{V} \cong 0$$

$$\frac{D\vec{V}}{Dt} = -\frac{1}{\rho} \vec{\nabla} p - 2\vec{\Omega} \times \vec{V} - \vec{\Omega} \times \vec{\Omega} \times \vec{r} + \vec{a}_g + \vec{a}_f$$

$$-\vec{\Omega} \times \vec{\Omega} \times \vec{r} = -\vec{\Omega} \times (\vec{\Omega} \|\vec{r}\| \cos \phi \hat{i}) =$$

$$-\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & |\vec{\Omega}| \cos \phi & |\vec{\Omega}| \sin \phi \\ (\vec{\Omega} \|\vec{r}\| \cos \phi \hat{i}) & 0 & 0 \end{vmatrix} =$$

$$-|\vec{\Omega}|^2 |\vec{r}| \cos \phi \sin \phi \hat{j} + |\vec{\Omega}|^2 |\vec{r}| \cos^2 \phi \hat{k}$$

$$-2\vec{\Omega} \times \vec{V} = -2 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & |\vec{\Omega}| \cos \phi & |\vec{\Omega}| \sin \phi \\ u & v & w \end{vmatrix} =$$

$$-2\vec{\Omega} [(w \cos \phi - v \sin \phi) \hat{i} + (u \sin \phi) \hat{j} - (u \cos \phi) \hat{k}]$$

$$\vec{G}_a = -G \frac{mM}{|r|^2} \hat{r}$$

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$$\vec{g}^* = \vec{g}_a - \vec{\Omega} \times \vec{\Omega} \times \vec{r}$$

$$v \nabla^2 u = \frac{1}{\rho} \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right)$$

$$v \nabla^2 v = \frac{1}{\rho} \left( \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \right)$$

$$v \nabla^2 w = \frac{1}{\rho} \left( \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} \right)$$

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -2\vec{\Omega}[(w \cos \phi - v \sin \phi)] - \frac{1}{\rho} \frac{\partial p}{\partial x} + v \nabla^2 u$$

$$\frac{Dv}{Dt} = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -2\vec{\Omega}[(u \sin \phi)] - \frac{1}{\rho} \frac{\partial p}{\partial y} + v \nabla^2 v$$

$$\frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -g - 2\vec{\Omega}[-(u \cos \phi)] - \frac{1}{\rho} \frac{\partial p}{\partial z} + v \nabla^2 w$$

$$-2\vec{\Omega}[(v_g \sin \phi)] \cong -\frac{1}{\rho} \frac{\partial p}{\partial x}; 2\vec{\Omega}[(u_g \sin \phi)] \cong -\frac{1}{\rho} \frac{\partial p}{\partial y}$$

$$\vec{V}_g = u_g \hat{i} + v_g \hat{j} = \hat{k} \times \frac{1}{\rho f} \vec{\nabla} p$$

$$-g \tan(\beta) = f V_g$$

$$[V_{g1} - V_{g2}] = \frac{1}{fL} (\bar{\alpha}_a - \bar{\alpha}_b)(P_1 - P_2)$$

$$Z \equiv \frac{1}{g_o} \int_0^z g \partial_z = \frac{\Phi}{g_o}$$

$$\left( -\frac{1}{\rho} \frac{\partial p}{\partial n} \right)_z = \left( -\frac{\partial \Phi}{\partial n} \right)_p$$

$$v_g = \frac{1}{f} \frac{\partial \Phi}{\partial x}; u_g = \frac{1}{f} \frac{\partial \Phi}{\partial y}$$

$$\vec{V}_T \equiv \vec{V}_g(p_1) - \vec{V}_g(p_o) = -\frac{R}{f} \int_{p_0}^{p_1} (k \times \vec{\nabla}_p T) \partial \ln p$$

$$u_T = -\frac{R}{f} \left( \frac{\partial \langle T \rangle}{\partial y} \right)_p \ln \left( \frac{p_o}{p_1} \right); v_T = \frac{R}{f} \left( \frac{\partial \langle T \rangle}{\partial x} \right)_p \ln \left( \frac{p_o}{p_1} \right)$$

$$-\frac{V^2}{r} - fV - \frac{\partial \Phi}{\partial n} = 0$$

$$V = -\frac{fr}{2} \pm \sqrt{\frac{f^2 r^2}{4} - r \frac{\partial \Phi}{\partial n}}$$

Actual = Mean + Perturbation

$$\sigma_a^2 = \overline{a' a'}$$

$$\sigma_a = \sqrt{\overline{a' a'}}$$

$$\sigma_{ab}^2 = \overline{a' b'}$$

$$r_{ab} = \frac{\overline{a' b'}}{\sigma_a \sigma_b}$$

$$\frac{\partial \bar{u}}{\partial t} + u \frac{\partial \bar{u}}{\partial x} + v \frac{\partial \bar{u}}{\partial y} + w \frac{\partial \bar{u}}{\partial z} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + f \bar{v} - \frac{\partial \bar{u}'^2}{\partial x} + \frac{\partial \bar{u}' \bar{v}'}{\partial y} + \frac{\partial \bar{u}' \bar{w}'}{\partial z}$$

$$\frac{\partial \bar{v}}{\partial t} + u \frac{\partial \bar{v}}{\partial x} + v \frac{\partial \bar{v}}{\partial y} + w \frac{\partial \bar{v}}{\partial z} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial y} - f \bar{u} - \frac{\partial \bar{u}' \bar{v}'}{\partial x} + \frac{\partial \bar{v}'^2}{\partial y} + \frac{\partial \bar{v}' \bar{w}'}{\partial z}$$

## Useful Values

Pressure conversions

$$1\text{mb} = 100\text{Pa} = 100 \text{ Nm}^{-2}$$

Molecular mass of dry air

$$m_a = 28.966$$

Molecular mass of water

$$m_w = 18.016$$

Universal gas constant

$$R^* = 8.31436 \text{ J mole}^{-1} \text{ K}^{-1}$$

Gas constant for dry air

$$R = R^*/m_a = 287.04 \text{ J kg}^{-1} \text{ K}^{-1}$$

Gas constant for water vapor

$$R_v = R^*/m_w = 461.50 \text{ J kg}^{-1} \text{ K}^{-1}$$

Molecular weight ratio

$$\epsilon = m_w/m_a = R_a/R_v = 0.62197$$

Rotation rate of earth

$$\Omega = 7.292 \times 10^{-5} \text{ s}^{-1}$$

Universal Gravitational Constant  
Mass of earth

$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$
$$M = 5.98 \times 10^{24} \text{ kg}$$

Radius of sphere having the same volume as the earth:

$$a = 6371 \text{ km}$$
$$\text{equatorial radius} = 6378 \text{ km}$$
$$\text{polar radius} = 6357 \text{ km}$$

## SCALING FACTORS FOR MID-LATITUDE SYNOPTIC SCALE MOTIONS

For Atmosphere:

$U \sim 10 \text{ m/sec}$	Horizontal velocity scale (typical u, v)
$W \sim 10^{-2} \text{ m/sec}$	Vertical velocity scale (typical w)
$L \sim 10^6 \text{ m}$	Horizontal length scale
$D \sim 10^4 \text{ m}$	Depth scale (typical depth of troposphere)
$\Delta p_h \sim 10^3 \text{ Pa}$	Horizontal pressure fluctuation scale
$\Delta p_v \sim 10^5 \text{ Pa}$	Vertical pressure fluctuation scale
$T_h = L/U \sim 10^5 \text{ sec}$	Horizontal time scale
$T_v = D/W \sim 10^6 \text{ sec}$	Vertical time scale
$g \sim 10 \text{ m/sec}^2$	Acceleration due to gravity
$\rho \sim 1 \text{ kg/m}^3$	Air density (lower troposphere)
$\mu/\rho \sim 10^{-5} \text{ m}^2/\text{sec}$	Kinematic molecular viscosity coefficient
$\Omega \sim 10^{-4} / \text{sec}$	earth's angular velocity

For Ocean:

$U \sim 10^{-1} \text{ m/sec}$	Horizontal velocity scale (typical u, v)
$W \sim 10^{-4} \text{ m/sec}$	Vertical velocity scale (typical w)
$L \sim 10^6 \text{ m}$	Horizontal length scale
$D \sim 10^3 \text{ m}$	Depth scale (typical depth of ocean)
$\Delta p_h \sim 10^4 \text{ Pa}$	Horizontal pressure fluctuation scale
$\Delta p_v \sim 10^7 \text{ Pa}$	Vertical pressure fluctuation scale
$T_h = L/U \sim 10^7 \text{ sec}$	Horizontal time scale
$T_v = D/W \sim 10^7 \text{ sec}$	Vertical time scale
$g \sim 10 \text{ m/sec}^2$	Acceleration due to gravity
$\rho \sim 1000 \text{ kg/m}^3$	Typical sea water density
$\mu/\rho \sim 10^{-6} \text{ m}^2/\text{sec}$	Kinematic molecular viscosity coefficient
$\Omega \sim 10^{-4} / \text{sec}$	earth's angular velocity

## SCALING FACTORS FOR OTHER MOTIONS

For Hurricanes:

$U \sim 10^2$ m/sec	Horizontal velocity scale (typical u, v)
$W \sim 10^0$ m/sec	Vertical velocity scale (typical w)
$L \sim 10^5$ m	Length scale (typical wavelength)
$H \sim 10^4$ m	Depth scale (typical depth of troposphere)
$\Delta p_h \sim 10^4$ Pa	Horizontal pressure fluctuation scale
$\Delta p_v \sim 10^5$ Pa	Vertical pressure fluctuation scale
$T_h = L/U \sim 10^3$ sec	Horizontal time scale (time required for a parcel to travel one wavelength)
$T_v = H/W \sim 10^4$ sec	Vertical time scale (time required for a parcel to travel through vertical depth scale distance)
$f \sim 10^{-4}$ sec <sup>-1</sup>	Coriolis parameter (mean value)
$g \sim 10$ m/sec <sup>2</sup>	Acceleration due to gravity
$\rho \sim 1$ kg/m <sup>3</sup>	Air density (lower troposphere)
$\mu/\rho \sim 10^{-5}$ m <sup>2</sup> /sec	Kinematic molecular viscosity coefficient

For Tornadoes:

$U \sim 10^2$ m/sec	Horizontal velocity scale (typical u, v)
$W \sim 10^2$ m/sec	Vertical velocity scale (typical w)
$L \sim 10^2$ m	Length scale (typical wavelength)
$H \sim 10^3$ m	Depth scale (typical depth of troposphere)
$\Delta p_h \sim 10^4$ Pa	Horizontal pressure fluctuation scale
$\Delta p_v \sim 10^4$ Pa	Vertical pressure fluctuation scale
$T_h = L/U \sim 10^0$ sec	Horizontal time scale (time required for a parcel to travel one wavelength)
$T_v = H/W \sim 10^1$ sec	Vertical time scale (time required for a parcel to travel through vertical depth scale distance)
$f \sim 10^{-4}$ sec <sup>-1</sup>	Coriolis parameter (mean value)
$g \sim 10$ m/sec <sup>2</sup>	Acceleration due to gravity

$$\rho \sim 1 \text{ kg/m}^3 \quad \text{Air density (lower troposphere)}$$
$$\mu/\rho \sim 10^{-5} \text{ m}^2/\text{sec} \quad \text{Kinematic molecular viscosity coefficient}$$